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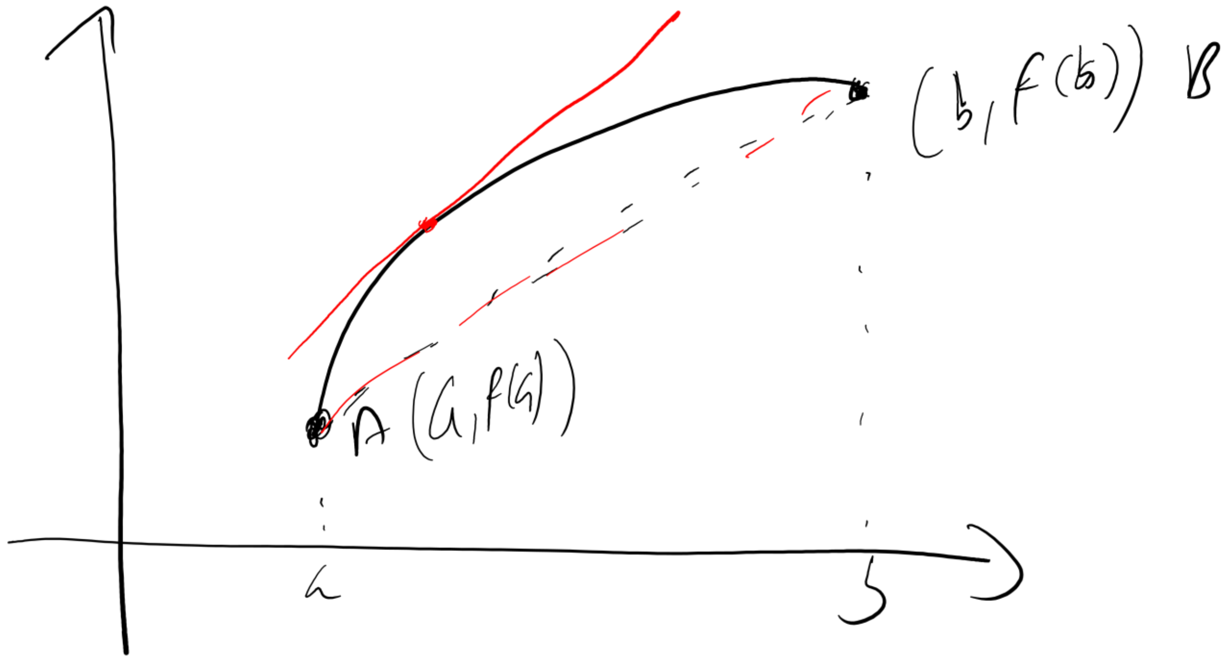
HSA MATHEMATICS

ATHUL S MURALI

Lagrange's Mean Value Theorem



Suppose that $y = f(x)$ is continuous over the closed interval $[a, b]$ and differentiable at every point of its interior (a, b). Then there is at least one point c in (a, b) at which

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}.}$$

$$\frac{f(b) - f(a)}{b-a} = m$$

$$\cancel{f'(x) = m}$$

i) f is cont. on $[a, b]$.

ii) f is diff on (a, b).

$\exists\, c \in (a, b)$

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

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Note

- **Rolle's theorem is a consequence of MVT**

$\exists\, c \in (a,b)$ s.t. $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$\rightarrow$ Rolle's $\boxed{f(a)=f(b)}$ $\rightarrow$ $f'(c) = \dfrac{f(b)-f(a)}{b-a} = \underline{\underline{0}}$

$\Rightarrow \exists\, c \in (a,b)$ s.t. $\underline{\underline{f'(c)=0}}$

Corollary

- If $f'(x) = 0$ at each point x of an open interval (a, b), then $f(x) = C\ \forall x \in (a, b)$, where C is a constant.

arbitrary

$(x_1, x_2) \subseteq (a, b)$

$f'(x) = 0 \ \forall x \in (x_1, x_2)$

$\exists\, c \in (x_1, x_2)$ s.t. $f'(c) = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$\Rightarrow \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = 0 \Rightarrow f(x_2) - f(x_1) = 0$

$\Rightarrow \boxed{f(x_2) = f(x_1)}$

$\boxed{f(x) = C}$

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Corollary

- If $f'(x) = g'(x)$ at each point x in an open interval (a, b), then there exists a constant C such that $f(x) = g(x) + C, \ \forall x \in (a, b)$. That is, $f - g$ is a constant function on (a, b).

$h(x) = f(x) - g(x) \Rightarrow h'(x) = f'(x) - g'(x) = 0$, $h'(x) = 0 \ \forall x \in (a, b)$

$\Longrightarrow h(x) = C \Rightarrow f(x) - g(x) = C$

$\Rightarrow \boxed{f(x) = g(x) + C}$

Find the value of c that satisfy MVT for

- $f(x) = x^2 + 2x - 1, \boxed{x \in [0, 1]}$

$\hookrightarrow$ poly f^n $\Rightarrow$ cont and diff L.M.V.T ✓

$f(0) = 0 + 2 \times 0 - 1 = -1$

$f(1) = 1 + 2 - 1 = 2$

$\exists c$ s $f'(c) = \dfrac{f(1) - f(0)}{1 - 0} = \dfrac{2 - -1}{1} = 3$

$\Rightarrow \boxed{f'(c) = 3}$

$f'(x) = 2x + 2$

$f'(c) = 3$

$\Rightarrow 2c + 2 = 3$

$\Rightarrow 2c = 1$

$\Rightarrow c = 1/2$

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- $f(x) = x + \frac{1}{x}, \quad x \in \left[\frac{1}{2}, 2\right]$

f is cont $[\frac{1}{2}, 2]$

diff on $(\frac{1}{2}, 2)$

$$f'(c) = \frac{f(2) - f(1/2)}{2 - 1/2} = \frac{2 + \frac{1}{2} - \left(\frac{1}{2} + 2\right)}{2 - 1/2} = 0$$

$\boxed{f'(c) = 0}$

$f'(x) = 1 - \frac{1}{x^2}$

$f'(c) = 0 \Rightarrow 1 - \frac{1}{c^2} = 0 \Rightarrow \frac{1}{c^2} = 1 \Rightarrow c^2 = 1 \Rightarrow \boxed{c = \pm 1}$

$c = \pm 1$

$c \in \left(\frac{1}{2}, 2\right)$

$\Rightarrow \boxed{c = 1}$

Which of the following functions satisfy the hypotheses of the Mean Value Theorem on the given interval

a) $f(x) = x^{\frac{2}{3}}, \quad x \in [-1, 8]$ ✗

b) $f(x) = \log x, \quad x \in \left[\frac{1}{2}, 2\right]$ ✓

a) $f(x) = x^{2/3}$ is cont on $[-1, 8]$

$$\boxed{f'(x) = \frac{2}{3} x^{-1/3} = \frac{2}{3x^{1/3}}}$$

at $x = 0$, $f'(x)$ does not exist

$$\lim_{h \to 0} \frac{h^{2/3} - 0}{h} = \lim_{h \to 0} h^{-1/3} \to \text{doesn't exist}$$

Not diff at $x = 0$

$\Rightarrow$

b) $f(x) = \log x$, $x \in \left[\frac{1}{2}, 2\right]$

↳ is cont

↳ is diff.

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Find the number of zeros of the following functions in the given interval.

a) $f(x) = x^4 + 3x + 1,\ [-2, -1]$

$f(x) = x^4 + 3x + 1 = 0$

$f(-2) = 16 - 6 + 1 = 11 > 0$

$f(-1) = 1 - 3 + 1 = -1 < 0$

$\Rightarrow$ f cuts x-axis at least once

$\Rightarrow$ f has zero on $[-2, -1]$

$f'(x) = 4x^3 + 3 < 0,\ x \in [-2, -1]$

$\Rightarrow$ f is strictly decreasing fn

$\Rightarrow$ f has only one zero

$$f(x) = x^3 + \frac{4}{x^2} + 7, \ (-\infty, 0) \longrightarrow 1$$

$f(-1) = -1 + 4 + 7 > 0$

$f(-3) = -27 + \frac{4}{9} + 7 < 0$

$\Rightarrow$ f has atleast one zero in $(-\infty, 0)$

$f'(x) = 3x^2 - \frac{8}{x^3} > 0$

$\downarrow$

$x \in (-\infty, 0)$

$x \in (-\infty, 0)$

$x^3 < 0$

$x^2 > 0$

$\Rightarrow$ f is Inc f^n

$\Rightarrow$ $\exists$ only one zero

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Cauchy's Mean Value Theorem

Suppose that f and g are continuous on $[a, b]$ and differentiable throughout (a, b) and also suppose $g'(x) \neq 0, \quad \forall x \in (a, b)$. Then there exist a number $c \in (a, b)$ at which

$$\frac{f'(c)}{g'(c)} = \frac{f(a) - f(b)}{g(b) - g(a)}.$$

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INVT

Cont [a,b]

Diff (a,b)

THANK YOU

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